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Refined c_f relation for turbulent channels and consequences for high-Re experiments

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Abstract

There have been rising concerns regarding the accuracy of measurements in turbulent channel flows, in particular the skin-friction results. In the present study, two different methods, namely, mean streamwise pressure gradient (PG) and oil film interferometry (OFI), are used to estimate the wall skin-friction relation, $c_f = f(Re_\mathrm{m})$, for fully developed turbulent plane-channel flow over a wide range of Reynolds numbers. The channel skin-friction data are then fitted to the well-known logarithmic friction law, providing outstanding agreement with values for the constants of the logarithmic law of the mean velocity profile. A revised logarithmic skin-friction relation is developed, providing good agreement with our skin-friction results and data from the literature, when constants of the logarithmic friction relation adopted from the recent work of Zanoun *et al* (2003 *Phys. Fluids* **15** 3079–89, 2005 *4th Int. Conf. on Heat Transfer, Fluid Mechanics and Thermodynamics, HEAT2005, 19–22 September, Cairo, Egypt*) are utilized. A new experimental channel facility is proposed, allowing measurements at high Reynolds numbers well beyond those achieved previously in laboratories, i.e. over five times the highest Re_m reached in the present study, while maintaining sufficiently high spatial resolution.

1. Introduction

Among several other recent publications, the first author's research work (Zanoun 2003, Zanoun *et al* 2002, 2003, 2005, 2007) has shown that errors in measured wall-shear stresses significantly impact the final scaling results in wall-bounded turbulent flows. For example,

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$\pm 1\%$ error in measuring the wall friction velocity results in an error of the same order in the so-called von Kármán constant of the logarithmic velocity profile. Accurate and independent determination of the wall-shear stress is therefore essential to establish reliable scaling of wall-bounded shear flows. Errors in determining u_τ are typically higher than those encountered in mean velocity measurements, and hence, inaccurate wall-shear stress measurements are usually the primary source of errors in estimating the constants of the logarithmic law of the wall. In the present paper, two independent methods are described to examine the wall-shear stress in fully developed turbulent plane-channel flows:

- The streamwise pressure gradient (PG) $(\mathrm{d}P/\mathrm{d}x)$ measured along the channel wall determines the wall-shear stress using a simple integral momentum balance

$$\tau_{\mathrm{w}} = -\frac{H}{2}\left(\frac{\mathrm{dP}}{\mathrm{dx}}\right), \qquad u_\tau = \sqrt{\frac{\tau_{\mathrm{w}}}{\rho}}. \tag{1}$$

Such measurements require the assumption that the mean flow statistics are two dimensional. Theoretically, the non-circular channel flow is considered two dimensional for high aspect ratios, $B/H \gg 1$, where B is the channel width and H the channel full height. Through a comprehensive review, Dean (1978) postulated that the minimum aspect ratio to ensure flow two dimensionality in channels is $7:1$, and recently Monty (2005) confirmed Dean's conclusions. On the other hand, Bradshaw and Hellens (1964) suggested a minimum aspect ratio of $5:1$ to assure the two dimensionality at the centerline of the channel. In the corner region of a duct of non-circular cross section, secondary flows are generated and these interact with both the axial mean velocity and the turbulent velocity fluctuations. As a result, deviations in pressure measurements along the centerline of a channel with a small aspect ratio are found in comparisons with measurements in two-dimensional channel flows of higher aspect ratios. Gessener and Jones (1965) and Gessener (1993) observed that the streamwise velocity component, even in the central region of the channel, is slightly affected by the convective transport associated with the secondary flow near the channel corners. This suggests that there may be an insignificant influence of the side wall on the mean flow properties along the centerline of channels having aspect ratios larger than $7:1$. Also, from an overall consideration of the mean flow and turbulence, it was suggested by Dean (1978) that three-dimensional effects become measurable only if the channel aspect ratio is less than $7:1$. Hence, based on the above arguments and a systematic investigation with various B/H ratios by Zanoun (2003), and to remove any questions regarding the two dimensionality of the mean flow, all measurements in the present study were taken for $B/H = 12:1$.

To obtain the skin friction from PG measurements, a plane-channel test section, see figure 1, described by Zanoun *et al* (2002, 2003) was utilized. The mean PG showed an earlier state of streamwise homogeneity at approximately $20H$ from the channel inlet test section. An inlet region of $30H$ length was found to be sufficient to ensure a fully developed state of the mean pressure distribution in the flow direction for the first measuring location; see e.g. Patel and Head (1969) and Comte-Bellot (1963). Therefore, we measured the mean PG over a distance of $85H$ starting at $30H$ and ending at $115H$ from the channel inlet. The last location for the mean pressure measurements was $15H$ away from the channel outlet, ensuring that there were no outlet disturbances to the investigated flows. For each flow case investigated, the bulk flow velocity ($\overline{U}$) was monitored at the channel entrance using a pitot probe. In addition, $\overline{U}$ was also obtained by integrating the velocity profile for each Reynolds number to ensure a good assessment of the bulk flow velocity. The bulk flow velocity was then used to compute the mean-based Reynolds number of the flow, $Re_{\mathrm{m}} = \overline{U}H/\nu$. A wide range of Reynolds numbers up to $Re_{\mathrm{m}} \approx 2.4 \times 10^5$ was achieved in the facility.

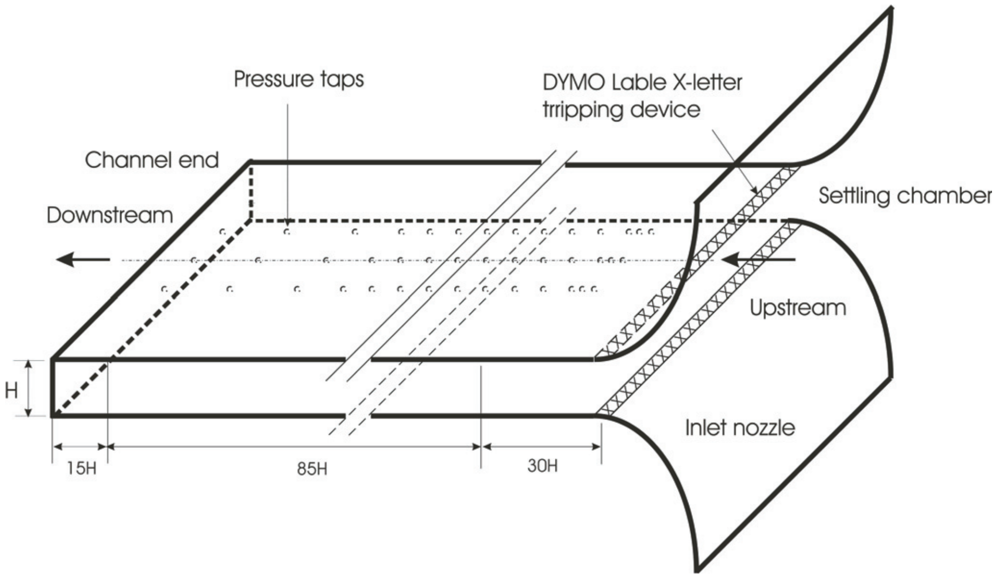

Figure 1. Sketch of the channel test section.

- An alternative technique to obtain the wall skin friction locally is the so-called oil film interferometry (OFI) (Durst *et al* 1996, Nagib *et al* 2004, Zanoun *et al* 2002). The principle of this technique is to follow the evolution of fringes that result from an interference pattern of a thin film illuminated by monochromatic light.

An oil film of thickness $h = h(x, t)$ is placed inside the channel test section where wall-shear stress measurements are desired. The oil film is then driven by the imposed wall-shear stress, $\tau_w(x, h, t)$, along its free surface, and consequently the shear stress can be evaluated by measuring $h(x, t)$. The variation of the oil film thickness in a two-dimensional shear flow along a surface is given by the following differential equation:

$$\frac{\partial h}{\partial t} + \frac{\tau_w h}{\mu_o}\frac{\partial h}{\partial x} = 0, \tag{2}$$

where h is the oil film thickness and μ_o the oil dynamic viscosity. Equation (2) reveals that to obtain a value of the wall-shear stress, it is necessary to measure h, $\partial h/\partial x$ and $\partial h/\partial t$. It shows also that the slope of h is constant in the x–t-plane along characteristic trajectories and the inverse slope of these trajectories, or contour lines, is

$$u_k = \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\tau_w h}{\mu_o}. \tag{3}$$

This leads one to focus on the rate of evolution of fringes, which are usually generated from the analysis of image stacks of the oil film captured (see figure 2). Consequently, the skin-friction information is obtained by solving equation (2) and yielding the wall-shear stress

$$\tau_w = \mu_o u_k \frac{2\left(n^2 - \sin^2\alpha\right)^{1/2}}{\lambda\left(k + (h_0/\Delta h)\right)} \Rightarrow c_f = \frac{\tau_w}{1/2\rho \overline{U}^2}. \tag{4}$$

Here, h_0 is the height of the zeroth black line in the fringe pattern, i.e. at the film edge (i.e. $k = 0$), h_k the height of the film at the kth black fringe, Δh $(= (\lambda/2)\sqrt{n^2 - \sin^2\alpha})$ is

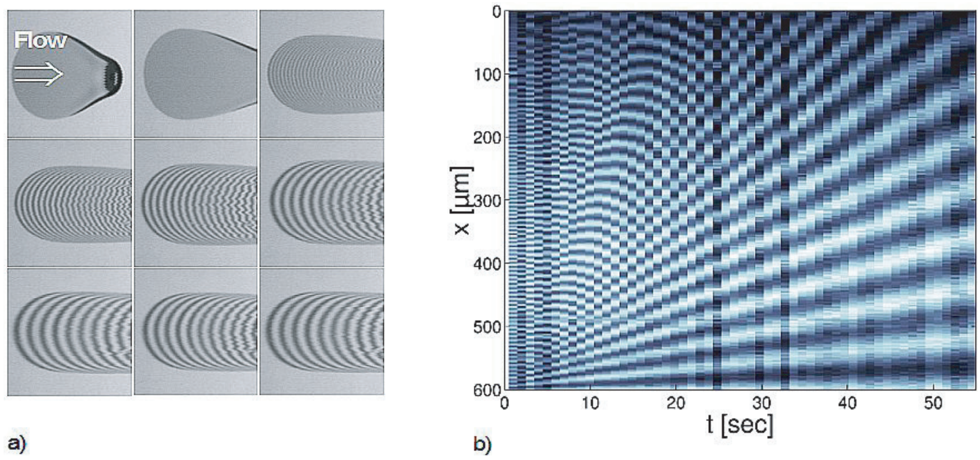

Figure 2. (a) Image stacks of an oil film droplet showing development of fringe pattern. (b) Oil film thickness development over the x–t diagram for a constant wall-shear stress.

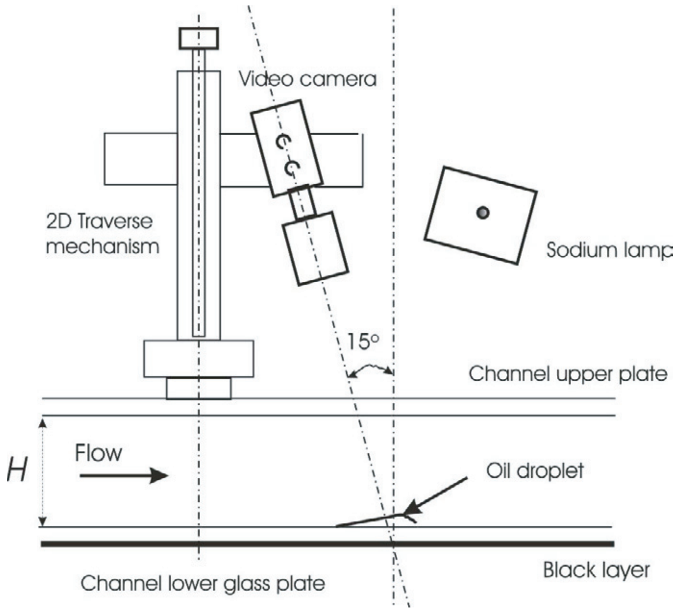

Figure 3. Optical setup of the oil film interferometry technique.

the height difference between two consecutive fringes, λ the light wavelength, n the refractive index of the oil and α the camera viewing angle. Values of μ_o, n, α and λ are usually known, and from the x–t diagram (figure 3) using an arbitrary number ($k = 10$) of fringe velocities, values of u_k are extracted. An error analysis using equation (4) to evaluate the accuracy of the oil film technique in the current setup found that it lies within ±2.4%, see Zanoun *et al* (2003) for more details. The major uncertainty in oil film measurements arises from the accuracy of the oil viscosity measurements. Hence, the oil viscosity measurements were performed carefully utilizing four different techniques (rotation rheometer, capillary tube viscometer, the cone plate and concentric cylinder rheometers). All values of viscosity measurements were close to each other, yielding an overall accuracy better than ±2%.

The optical setup for the OFI is shown in figure 3. The measuring equipment primarily consisted of a video camera that had an 8 mm black-band white CCD chip with a resolution of 750×770 pixels. The camera was equipped with two zoom lenses and a telescopic cover between them to prevent light reaching the CCD from the sides. A 30 W sodium lamp having a wavelength of 589 nm was used to illuminate the oil film region. A two-dimensional traversing mechanism was used for the video camera to obtain well-focused images of the oil film development and to record the resultant interference fringes. The angle between the camera axis and the normal direction to the glass wall forming the lower channel wall was 15° and was kept constant during all measurements. The glass plate of the channel test section was blackened on its lower side to provide a suitable surface for light reflection. A PCI frame grabber card was used to digitize the analog output of the camera at preset time intervals. A sequence of images (image stacks) as shown in figure 2 was recorded together with other parameters such as oil refractive index, light wavelength, the camera viewing angle and the oil temperature that was employed to define the oil viscosity. All information was then used for further processing of the video camera records to obtain the wall-shear stress.

2. Results

Utilizing carefully drilled small holes along the channel upper wall, mean pressure measurements were performed and then used to evaluate the streamwise PG, which in turn was employed to obtain the wall-shear stress and the wall friction velocity, u_τ, from equation (1). In addition to PG measurements, local wall skin-friction data were obtained from the OFI as described earlier. Evaluations of the wall skin-friction coefficient, c_f, were then established based on both techniques, and the results obtained are presented in figure 4 in the form of c_f as a function of Re_m. The wall skin-friction coefficient is calculated using the following equation:

$$c_f = 2\left(\frac{u_\tau}{\overline{U}}\right)^2. \tag{5}$$

As might be expected for channels of high enough aspect ratios, the wall-skin friction estimates obtained from PG measurements compared well with the local skin-friction measurements using the OFI as figure 4 demonstrates.

To proceed further, an empirical power relation is fitted to all of our skin-friction data, resulting in the following correlation:

$$c_f = 0.0743\, Re_\mathrm{m}^{-0.25}, \tag{6}$$

where Re_m is based on the channel full height and the bulk flow velocity. Equation (6) is found to deviate by about 1.8% from Dean's equation

$$c_f = 0.073\, Re_\mathrm{m}^{-0.25}. \tag{7}$$

It is worth noting here that Dean's relation (7) for the skin-friction coefficient was a result of fitting to a number of different experimental data extracted from the literature, in addition to his own data (Dean 1978). A wide scatter was apparent in the data for the skin-friction coefficient shown in figure 2 in Dean's paper. Such an uncertainty in the data may have resulted in the smaller value of the coefficient in equation (7). One important factor that might have also contributed to the wide scatter in the c_f values found in the literature could be attributed to the method of determining the bulk flow velocity in the channel. It was observed that utilizing the bulk flow velocity from only one measurement at the channel

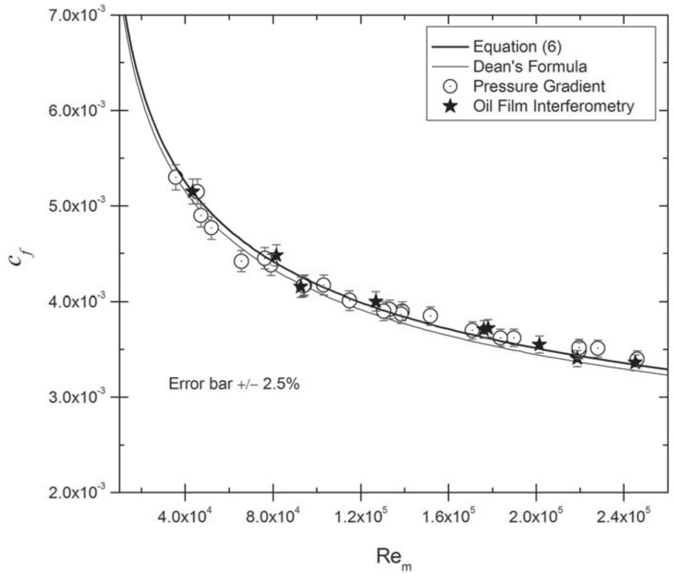

Figure 4. Present skin-friction data compared with Dean's relation (7) and equation (6).

entrance where the velocity distribution is assumed to be uniform is not completely correct, particularly for channels with small aspect ratios. The inconsistencies in the c_f data from the literature may also be due to other influencing factors, such as determining the wall shear stress in an undeveloped flow region near the inlet of the channel; see e.g. Laufer (1951) and Skinner (1951). In addition to channel aspect ratio, the wall skin friction is also found to depend on the degree of wall roughness, accuracy of measuring devices and temperature drift. However, the experimental conditions were carefully checked throughout the experimental measurements, and therefore the present data are considered to be of high accuracy.

3. The logarithmic skin-friction relation

The deviation of the mean velocity profile from the log law in the core region of a channel flow is much weaker than that for the pipe flow, see Zanoun *et al* (2005), and therefore its effect on estimating the mass flux is insignificant. This suggests that the log law may be approximately valid close to the line of symmetry of the channel flow and consequently it may be used for obtaining the bulk flow velocity, without significant errors. Hence, utilizing the logarithmic velocity profile, $U/u_\tau = (1/\kappa)\ln(yu_\tau/\nu) + A$, alone, and since the wall region contributions are also small and negated by the core region contributions, the bulk flow velocity may be estimated as follows:

$$\overline{U} = \frac{1}{BH}\int_0^H BU\,\mathrm{d}y. \tag{8}$$

With the above approximations, this relation may be written in the form

$$\frac{\overline{U}}{u_\tau} = \frac{1}{\kappa}\ln\left(\frac{(1/2)Hu_\tau}{\nu}\right) - \frac{1}{\kappa} + A, \tag{9}$$

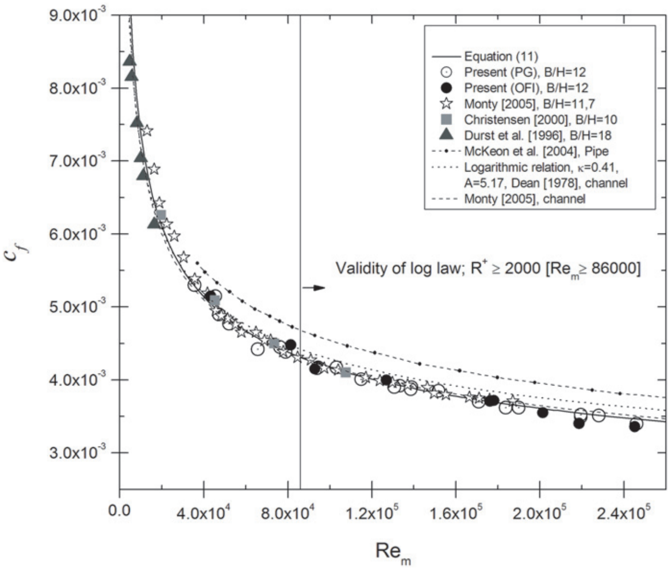

Figure 5. Representation of the logarithmic skin-friction relation, equation (11), compared with the present experimental results and data extracted from the literature.

where κ is the von Kármán constant and A an additive constant. Introducing the definition of the skin-friction coefficient, i.e. equation (5), into equation (9) results in the logarithmic friction relation for flow in channels as a function of Kármán number, $R^+ = (1/2)Hu_\tau/\nu$, or the mean-based Reynolds number, Re_m, and can be written in the following forms:

$$\sqrt{\frac{2}{c_f}} = \frac{1}{\kappa}\ln\left(R^+\right) - \frac{1}{\kappa} + A \quad \text{or} \quad \sqrt{\frac{2}{c_f}} = \frac{1}{\kappa}\left[\ln\left(\frac{Re_\mathrm{m}}{2}\sqrt{\frac{c_f}{2}}\right) - 1\right] + A. \tag{10}$$

Utilizing the log law constants that were deduced from mean velocity profiles by Zanoun *et al* (2003, 2005), i.e. $\kappa = 0.37$ and $A = 3.7$, equation (10) can be rewritten as

$$\frac{1}{\sqrt{c_f}} = 1.911\,\ln\left(Re_\mathrm{m}\sqrt{c_f}\right) - 1.282. \tag{11}$$

Figure 5 shows c_f obtained from equation (11) compared with all of our data. In addition, data of Monty (2005) for a channel flow are represented in figure 5. The skin-friction data of Monty (2005) were obtained from PG measurements in a channel with an aspect ratio of 11.7:1 and for $Re_\mathrm{m} < 10^5$. We have also included data from Christensen (2001), and some from Durst *et al* (1996) for the low Reynolds number range. Good agreement is observed between the different experimental results and the logarithmic skin-friction relation proposed, i.e. equation (11), in particular, for $R^+ \geqslant 2000$. In addition, a logarithmic relation proposed by Monty (2005) is presented in figure 5 showing good agreement with equation (11). On the other hand, a logarithmic relation employing the constants proposed by Dean (1978), i.e. $\kappa = 0.41$ and $A = 5.17$, exhibits less agreement with data in the high Reynolds number range. For contrast, and to emphasize the differences between fully developed channel and pipe flows, we have also included the recent logarithmic relation for pipes by McKeon *et al* (2004), which extends to very high Reynolds numbers starting from $Re_\mathrm{m} \geqslant 300 \times 10^3$, where Re_D is based on the pipe diameter and bulk flow velocity.

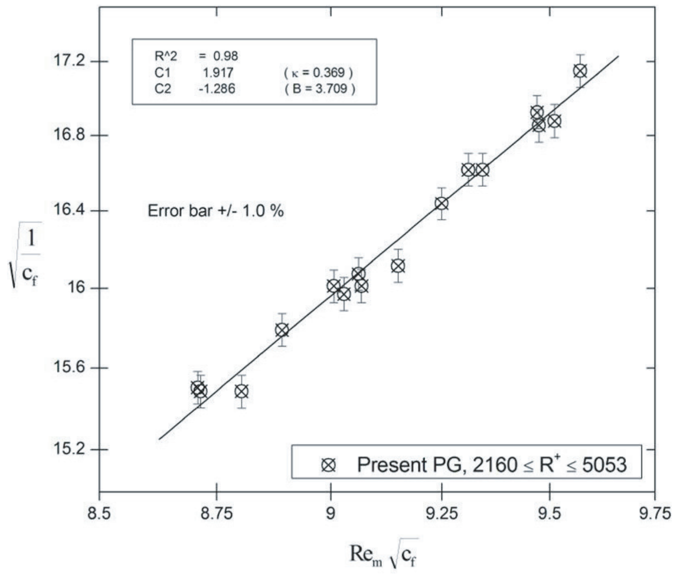

Figure 6. Mean PG skin-friction data fitted to the logarithmic skin-friction relation (12).

On the other hand, equation (10) may be directly used to evaluate both constants of the logarithmic law of the wall, i.e. κ and A, after rearranging it to take the following simplified form:

$$\sqrt{\frac{1}{c_f}} = C_1 \ln(Re_{\mathrm{m}}\sqrt{c_f}) + C_2. \tag{12}$$

where

$$C_1 = \frac{1}{\sqrt{2}\,\kappa} \quad \text{and} \quad C_2 = \frac{1}{\sqrt{2}}\left[A - \frac{1}{\kappa}\left(1 + \ln(2\sqrt{2})\right)\right].$$

C_1 and C_2 are coefficients that may or may not be Reynolds number dependent. Therefore, equation (12) can be used to determine κ without the need to evaluate the slope of the mean velocity profile over a limited range of the wall distance. It is worth mentioning that the logarithmic skin friction law was deduced based on complete similarity of the mean velocity profile in both the inner and outer flow regions, similar to that of Prandtl (1933) for pipe flow. Therefore, this is an interesting method to estimate both constants κ and A from only integral flow parameters such as the bulk flow velocity and the mean PG, however, only for high enough Reynolds number, i.e. $R^+ \geqslant 2000$.

The present channel skin-friction data were further analyzed with respect to the question of whether the logarithmic friction relation can be used to derive accurately the constants of the log law or not. From the analysis of the channel mean velocity data by Zanoun *et al* (2003), the logarithmic velocity profile was found to be valid and Re independent for $R^+ \geqslant 2000$ (i.e. $Re_{\mathrm{m}} \geqslant 8.6 \times 10^4$). Therefore, the present friction data were plotted versus the Reynolds number and fitted to equation (12) in figure 6 for the higher range of Reynolds number data, i.e. $8.6 \times 10^4 \leqslant Re_{\mathrm{m}} \leqslant 2.46 \times 10^5$, to determine both C_1 and C_2. A summary of both constants, C_1 and C_2, and the corresponding values of κ and A are given in table 1. From PG skin-friction data for $9.388 \times 10^4 \leqslant Re_{\mathrm{m}} \leqslant 2.46 \times 10^5$ ($2160 \leqslant R^+ \leqslant 5053$), values

Table 1. Summary of constants of the logarithmic friction relation and logarithmic velocity profile utilizing equation (12).

Present (PG) $2160 \leqslant R^+ \leqslant 5053$		Present (OFI) $2113 \leqslant R^+ \leqslant 4514$	
C_1 (κ)	C_2 (A)	C_1 (κ)	C_2 (A)
1.917 (0.369)	−1.286 (3.71)	1.981 (0.36)	−1.852 (3.1)

of both $\kappa = 0.369$ and $A = 3.71$ were found to be in good agreement with values obtained by the present authors based on the slope of the mean velocity profiles ($\kappa = 0.37$ and $A = 3.7$), see Zanoun *et al* (2003). Contrary to the values extracted using the skin-friction data based on the PG, the local skin-friction data obtained from OFI for $9.28 \times 10^4 \leqslant Re_\mathrm{m} \leqslant 2.186 \times 10^5$ ($2113 \leqslant R^+ \leqslant 4514$) fitted to equation (11) result in lower values for both constants (i.e. $\kappa = 0.357 \cong 0.36$ and $A = 3.1$). The lower values obtained from the OFI raises a concern regarding whether the channel side walls have a small influence on the skin-friction data along the channel centerline. It is worth noting here that the Reynolds number effects on the wall-shear stress for channels of small aspect ratios remain uncertain and hence an accurate evaluation of the side wall contributions requires channels of varying aspect ratio and further experimental analysis. In addition, there is not a rigorous criterion to establish an exact entrance length for the fully developed flow in a channel. Hence, it is very difficult to provide a good baseline for answering questions regarding side wall effects and dependence on transition to turbulence along a smooth and/or rough channel flow, or the role they play for different Reynolds numbers in the development length i.e. many unanswered questions and subjects of vital interest to the turbulence community.

4. Conclusions and outlook

From the results and discussions, we can conclude that recently measured skin-friction data for channel flows are found to be consistent with Dean's skin-friction relation (7) and well represented by the slightly refined empirical power relation $c_f = 0.0743\, Re_\mathrm{m}^{-0.25}$. However, a revised logarithmic skin-friction relation (11), with significantly different and properly chosen parameters, is even more accurate in representing the experimental wall skin-friction data (an accuracy better than $\pm 1\%$). On the other hand, we find that utilizing the same relation (12) to estimate the constants of the logarithmic velocity profile is very sensitive to small errors in skin-friction data.

In closing we would like to draw attention to some of the recent discussions arguing for a desired high-Reynolds number wall-turbulence experiment and its underlying challenges; see e.g. Talamelli *et al* (2008). High-Reynolds number experiments for the three canonical wall-bounded flows, i.e. pipes, channels and boundary layers, are important to researchers for fundamental understanding of turbulence theory and for many practical needs of turbulence modeling. In the case of pipes and boundary layer flows, various existing laboratory facilities have been successful in achieving Reynolds numbers high enough to address similarity of mean flow and scaling. For example, the pipe flow experiments of Zagarola and Smits (1998) achieved a Kármán number, $R^+ \approx 500 \times 10^3$, while in the experiments of Nagib *et al* (2004), $\delta^+ \approx 20 \times 10^3$ has been obtained in a well-documented laboratory boundary layer. In contrast, for the channel flow experiments, the highest Kármán number, R^+, achieved is comparatively smaller, i.e. of the order of 5×10^3. At this Reynolds number, the separation between the inner and outer scales is not large enough to study the desired characteristics of turbulence. Similar to the efforts made toward a high-Re_m pipe flow experiment with good spatial resolution (Talamelli *et al* 2008), increasing the maximum achievable R^+ for channel flow experiments is highly desirable. Therefore, we will outline here some of the desired characteristics and requirements of such a high-Re_m channel flow experiment.

Q2 Based on available DNS and experimental results, one finds that a Reynolds number-independent behavior and a well-defined overlap region are only achieved beyond an R^+ value of 5×10^3 or 6×10^3. Considering a new facility allows one to increase the Reynolds number so that it would extend over a decade in Reynolds number beyond these conditions, so a maximum Kármán number $R^+ \approx 50 \times 10^3$ is desired. From equation (9), the ratio of bulk velocity to the friction velocity can be estimated as $\overline{U}/u_\tau \approx 30$, resulting in a maximum desirable bulk velocity Reynolds number of $Re_\mathrm{m} \approx 3 \times 10^6$. Air at ambient temperature and pressure ($\nu \approx 1.5 \times 10^{-5}$) and limiting the maximum velocity to around 60 or 80 m s^{-1} in order to avoid compressibility effects yield an estimate for the channel height of around $H = 0.75$ or 0.55 m (channel half-height of 0.375 or 0.275 m), respectively. Assuming that an aspect ratio of $B/H = 12$ is sufficient for two dimensionality, one would need a channel duct that is 9 or 6.7 m wide, respectively. In contrast to a pipe flow with a similar diameter as the channel height, this area requirement is around 15 times larger (Talamelli *et al* 2008). Therefore, the power required to drive more than 300 m^3 s^{-1} through the channel would also be proportionally high. This gives one an idea why experimentalists have not so far been able to achieve large Reynolds numbers for channel flows in typical research laboratories. Hence, to perform high-Reynolds number channel flow experiments that would be useful to extend the present theoretical understanding, one needs a channel of at least 0.25–0.30 m half-height and 6–7.2 m width. For measurement in fully developed flow, an $L/H \approx 150$ will require a channel length between 75 and 90 m, i.e. a very large laboratory environment. Along with increasing Reynolds number, considerations for data acquisition and measurement setup should also be made. An outline for probe requirements and acquisition of higher turbulence statistics can be found in the recent work of Talamelli *et al* (2008).

A channel flow facility with high aspect ratio, and an adequate development length, capable of higher Re_m, while maintaining good measurement resolution, would facilitate research on a number of fundamental issues. The facility must provide for precise and independent wall skin-friction measurement to remedy the unsatisfactory situation found in the literature, e.g. see Abe *et al* (2004). Some of the fundamental topics that can be studied in such a facility include:

- Dynamics of laminar-to-turbulence transition and evolution of the mean and higher order statistics, and the frictional loss along the flow development length. This would allow examination of any relations that may exist between transition in duct flows and the scales at high Reynolds numbers.
- Turbulence scaling laws and scaling parameters for heat and momentum transport as well as their dependence on geometry of flow, e.g. compared with pipes and boundary layers under various PGs. Most turbulence models require wall functions based on the asymptotic behavior of the mean velocity distribution in the inertial sublayer. Documenting and understanding the similarity and differences with other flow geometries are of vital importance for a better understanding of wall-bounded shear flows.
- Some similarities and differences, in particular, the character of the outer layer (i.e. wake region), among three types of wall-bounded shear flows (pipe, channel and boundary layer) require new pipe and channel experiments.

- The role of surface roughness can be more effectively examined in a high-Reynolds number channel flow.

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References

Abe H, Kawamura H and Matsuo Y 2004 Surface heat-flux fluctuations in a turbulent channel flow up to $Re_\tau = 1020$ with $Pr = 0.025$ and 0.71 *Int. J. Heat Fluid Flow* **25** 404–19

Bradshaw P and Hellens G E 1964 The N.P.L. 59 in $\times$ 9 in boundary layer tunnel *NPL Aero Report 1119*

Comte-Bellot G 1963 Turbulent flow between two parallel walls *PhD Thesis* (in French) University of Grenoble (in English as ARC 31609), FM 4102

Christensen K T 2001 Experimental investigation of acceleration and velocity fields in turbulent channel flow *PhD Thesis* Department of Theoretical and Applied Mechanics, University of Illinois at Urbana-Champaign, IL, USA

Dean R B 1978 Reynolds number dependence of skin friction and other bulk flow variables in two-dimensional rectangular duct flow *ASME J. Fluid Eng.* **100** 215–23

Durst F, Kikura H, Lekakis I, Jovanović J and Ye Q-Y 1996 Wall shear stress determination from near-wall mean velocity data in turbulent pipe and channel flows *Exp. Fluids* **20** 417–28

Fernholz H H, Janke G, Schober M, Wagner P M and Warnack D 1996 New developments and applications of skin-friction measuring technique *Meas. Sci. Technol.* **77** 1396–409

Gessener F B and Jones J B 1965 On some aspects of fully-developed turbulent flow in rectangular channels *J. Fluid Mech.* **23** 689–713

Gessener F B 1993 Reynolds number effects on the near-wall structure of turbulent flow along a streamwise corner *Near Wall Turbulent Flows* ed R M C So, C G Speziale and B E Launder (Amsterdam: Elsevier) pp 965–76

Laufer J 1951 Investigation of turbulent flow in a two-dimensional channel *NACA Rep.* **1053** 1247–66

McKeon B J, Swanson C J, Zagarola M V, Donnelly R J and Smits A J 2004 Friction factors for smooth pipe flow *J. Fluid Mech.* **511** 41–4

Monty J P 2005 Developments in smooth wall turbulent duct flows *PhD Thesis* The University of Melbourne, Australia

Nagib H, Christophorou C and Monkewitz P 2004 High Reynolds number turbulent boundary layers subjected to various pressure-gradient conditions *IUTAM Symp. on One Hundred Years of Boundary Layer Research, DLR Göttingen, Germany* ed G Meier and K Sreenivasan (Berlin: Springer) pp 383–94

Patel V C and Head M R 1969 Some observations on skin friction and velocity profiles in fully developed pipe and channel flows *J. Fluid Mech.* **38** 181–201

Prandtl L 1933 *Z. Ver. Dtsch. Ing.* **77** 105 (Engl. Transl. NACA TM 720)

Rüedi J-D, Nagib H, Österlund J and Monkewitz P 2003 Evaluation of three techniques for wall-shear measurements in three-dimensional flows *Exp. Fluids* **35** 389–96 Q3

Skinner G T 1951 Mean speed measurements in two dimensional incompressible fully-developed turbulent channel flow *MS Thesis* California Institute of Technology

Talamelli A, Persiani F, Fransson J H M, Alfredsson H, Johansson A V, Nagib H M, Rüedi J-D, Sreenivasan K R and Monkewitz P A 2007 CICLoPE — a response to the need for high Reynolds number experiments *Fluid Dyn. Res.* to appear Q3

Tanner L H and Blows L G 1976 A study of the motion of oil films on surface in air flow, with application to the measurements of skin friction *J. Phys. E: Sci. Instrum.* **9** 194–202

Zanoun E-S, Nagib H, Durst F and Monkewitz P 2002 Higher Reynolds number channel data and their comparison to recent asymptotic theory *AIAA-2002-1102, 40th AIAA Aerospace Sciences Meeting and Exhibit* (*Reno, NV, 14–17 January 2002*)

Zanoun E-S, Durst F and Nagib H 2003 Evaluating the law of the wall in two-dimensional fully developed turbulent channel flows *Phys. Fluids* **15** 3079–89

Zanoun E-S 2003 Answers to some open questions in wall-bounded laminar and turbulent shear flows *PhD Thesis* Universität Erlangen Nürnberg

Zanoun E-S, Durst F and Nagib H 2005 Scaling laws for turbulent channel and pipe flows over a wide range of Reynolds number *4th Int. Conf. on Heat Transfer, Fluid Mechanics and Thermodynamics, HEAT2005* (*19–22 September 2005 Cairo, Egypt*)

Zanoun E-S, Durst F, Bayoumi O and Al-Salaymeh A 2007 Wall skin friction and mean velocity profiles of fully developed turbulent pipe flows *Exp. Therm. Fluid Sci.* **32** 249–61

Zagarola M V and Smits A J 1998 Mean-flow scaling of turbulent pipe flow *J. Fluid Mech.* **373** 33–79

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